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On the extensions of the Darboux theory of integrability

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Abstract
Recently some extensions of the classical Darboux integrability theory to autonomous and non-autonomous polynomial vector fields were completed. The classical Darboux integrability theory and its recent extensions are based on the existence of algebraic invariant hypersurfaces. However the algebraicity of the invariant hypersurfaces is not necessary and the unique necessary condition is the algebraicity of the cofactors associated to them. In this paper a more general extension of the classical Darboux integrability theory is established.

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1. Introduction

One of the classical problems in the theory of differential equations is to decide when a differential system is integrable or not. There is not a unique definition of integrability for a differential equation. Initially, the notion of 'integrability' was introduced to describe the property of differential equations for which all local and global information can be obtained either explicitly from solutions or implicitly from invariants. The first class of invariants are the constants of motion, conserved quantities or first integrals. Of course there are also other invariants like integral invariants, integrating factors, Jacobi multipliers, Lax pairs or Lax operators, tensor invariants, or symmetries which give rise to different techniques for integrating differential equations, see for instance [1, 4, 20, 22, 29, 31] and references therein. The connections between these different techniques are, in general, unknown and these connections encourage very active research, see for example [2, 6, 19].

