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J. Differential Equations 197 (2004) 147-161

Journal of Differential Equations

http://www.elsevier.com/locate/jde

## Integrability and algebraic limit cycles for polynomial differential systems with homogeneous nonlinearities $\stackrel{\text{tr}}{\rightarrow}$

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Received January 17, 2003; revised April 11, 2003

Dedicated to Professor George Sell's on the occasion of his 65th birthday

## Abstract

We consider the class of polynomial differential equations  $\dot{x} = \lambda x - y + P_n(x, y)$ ,  $\dot{y} = x + \lambda y + Q_n(x, y)$ , where  $P_n$  and  $Q_n$  are homogeneous polynomials of degree *n*. These systems have a focus at the origin if  $\lambda \neq 0$ , and have either a center or a focus if  $\lambda = 0$ . Inside this class we identify a new subclass of Darbouxian integrable systems having either a focus or a center at the origin. Additionally, under generic conditions such Darbouxian integrable systems can have at most one limit cycle, and when it exists is algebraic. For the case n = 2 and 3, we present new classes of Darbouxian integrable systems having a focus.  $\bigcirc$  2003 Elsevier Inc. All rights reserved.

MSC: Primary 34C35, 34D30

Keywords: Integrability; Algebraic limit cycle; focus; Center

 $<sup>^{\</sup>pm}$  The first author is partially supported by a MCYT grant number BFM 2002-04236-C02-01, by a University of Lleida Project P01 and by DURSI of Government of Catalonia's Acció Integrada ACI2001-26. The second author is partially supported by a DGICYT grant number BFM 2002-04236-C02-02 and by a CICYT grant number 2001SGR00173.

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