

Available online at www.sciencedirect.com



An International Journal computers & mathematics with applications



Computers and Mathematics with Applications 51 (2006) 1453-1462

www.elsevier.com/locate/camwa

Integrability, Degenerate Centers, and Limit Cycles for a Class of **Polynomial Differential Systems**

J. GINÉ

Departament de Matemàtica Universitat de Lleida, Av. Jaume II, 69, 25001 Lleida, Spain gine@eps.udl.es

J. LLIBRE

Departament de Matemàtiques Universitat Autònoma de Barcelona 08193 Bellaterra, Barcelona, Spain jllibre@mat.uab.es

(Received May 2005; revised and accepted January 2006)

Abstract—We consider the class of polynomial differential equations $\dot{x} = P_n(x, y) + P_{n+1}(x, y)$ $+P_{n+2}(x,y), \dot{y} = Q_n(x,y) + Q_{n+1}(x,y) + Q_{n+2}(x,y), \text{ for } n \ge 1 \text{ and where } P_i \text{ and } Q_i \text{ are homogeneous}$ polynomials of degree *i*. These systems have a linearly zero singular point at the origin if $n \geq 2$. Inside this class, we identify a new subclass of Darboux integrable systems, and some of them having a degenerate center, i.e., a center with linear part identically zero. Moreover, under additional conditions such Darboux integrable systems can have at most one limit cycle. We provide the explicit expression of this limit cycle. © 2006 Elsevier Ltd. All rights reserved.

Keywords-Integrability, Algebraic limit cycle, Linearly zero singular point, Degenerate center, Polynomial vector field, Polynomial differential system.

1. INTRODUCTION AND STATEMENT OF THE RESULTS

Probably the three main open problems in the qualitative theory differential systems in \mathbb{R}^2 are the determination of the number of the limit cycles and their distribution in the plane (see, for instance, [1]); the distinction between a center and a focus, called the *center problem* (see, for instance, [2]); and the determination of their first integrals (see, for instance, [3]). This paper deals with these three problems for a class of polynomial differential systems. More explicitly,

The first author is partially supported by a DGICYT grant number MTM2005-06098-C02-02 and by a CI-CYT grant number 2005SGR 00550, and by DURSI of Government of Catalonia "Distinció de la Generalitat de Catalunya per a la promoció de la recerca universitària". The second author is partially supported by a DGICYT grant number MTM2005-06098-C02-01 and by a CICYT grant number 2005SGR 00550.