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The flow curvature method applied to canard explosion

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Abstract

The aim of this work is to establish that the bifurcation parameter value leading to a *canard explosion* in dimension 2 obtained by the so-called *geometric singular perturbation method* can be found according to the *flow curvature method*. This result will be then exemplified with the classical Van der Pol oscillator.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The classical geometric theory of differential equations developed originally by Andronov [1], Tikhonov [30] and Levinson [23] stated that *singularly perturbed systems* possess *invariant manifolds* on which trajectories evolve slowly, and toward which nearby orbits contract exponentially in time (either forward or backward) in the normal directions. These manifolds have been called asymptotically stable (or unstable) *slow invariant manifolds*[3]. Then, Fenichel [11–14] theory[4] for the *persistence of normally hyperbolic invariant manifolds* enabled us to establish the *local invariance* of *slow invariant manifolds* that possess both expanding and contracting directions and which were labeled *slow invariant manifolds*.

During the last century, various methods have been developed to compute the *slow invariant manifold* or, at least an asymptotic expansion in the power of ε.

The seminal works of Wasow [32], Cole [6], O'Malley [25, 26] and Fenichel [11–14] to name a few, gave rise to the so-called *geometric singular perturbation method*. According to this theory, the existence as well as the local invariance of the *slow invariant manifold* of *singularly perturbed systems* has been stated. Then, the determination of the *slow invariant*

[3] In other articles, the *slow manifold* is the approximation of order $O(\varepsilon)$ of the *slow invariant manifold*.
[4] The theory of invariant manifolds for an ordinary differential equation is based on the work of Hirsch *et al* [19].

