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ON THE CENTER CONDITIONS FOR ANALYTIC MONODROMIC DEGENERATE SINGULARITIES

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In this paper, we present two methods for detecting centers of monodromic degenerate singularities of planar analytic vector fields. These methods use auxiliary symmetric vector fields and can be applied independently so that the singularity is algebraic solvable or not, or has characteristic directions or not. We remark that these are the first methods which allow to study monodromic degenerate singularities with characteristic directions.

Keywords: Degenerate center; first integral; Bautin method; algebraically solvable.

1. Introduction and Statement of the Main Results

One of the classical problems in the qualitative theory of planar analytic differential systems in $\mathbb{R}^2$ is to characterize the local phase portrait near an isolated singular point. By using the blow-up technique, this local phase portrait can be characterized for all the isolated singularities except for the monodromic singularities. We recall that a singularity is *monodromic* if there exists a neighborhood of the singularity such that the solutions of the differential equation turn around it either in forward or in backward time. For more details, see for instance [Dumortier *et al.*, 2006; García *et al.*, 2006] and references therein.

When the differential system is analytic, a monodromic singular point is either a center or a focus, see for instance [Anosov *et al.*, 1997]. A *center* is a singular point having a neighborhood filled with periodic orbits except by the singular point, and a *focus* is a singular point such that all the near orbits with the exception of the singular point spiral either in forward or in backward time to the singular point. The *center problem* consists on distinguishing between a center and a focus.

Let $p \in \mathbb{R}^2$ be a singular point of an analytic differential system in $\mathbb{R}^2$, and assume that p is a center. We suppose that p is at the origin of coordinates, otherwise by a translation of coordinates we send p to the origin. Then, after a linear change of variables and a rescaling of the time variable (if necessary), the system can be written in one of the following three forms:

$$\dot{x} = -y + F_1(x,y), \quad \dot{y} = x + F_2(x,y); \tag{1}$$

$$\dot{x} = y + F_1(x,y), \quad \dot{y} = F_2(x,y); \tag{2}$$

$$\dot{x} = F_1(x,y), \quad \dot{y} = F_2(x,y); \tag{3}$$

where $F_1(x,y)$ and $F_2(x,y)$ are real analytic functions without constant and linear terms, defined in