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Research Article

Canards Existence in FitzHugh-Nagumo and Hodgkin-Huxley Neuronal Models

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In a previous paper we have proposed a new method for proving the existence of "canard solutions" for three- and four-dimensional singularly perturbed systems with only one *fast* variable which improves the methods used until now. The aim of this work is to extend this method to the case of four-dimensional singularly perturbed systems with two *slow* and two *fast* variables. This method enables stating a unique generic condition for the existence of "canard solutions" for such four-dimensional singularly perturbed systems which is based on the stability of *folded singularities* (*pseudo singular points* in this case) of the *normalized slow dynamics* deduced from a well-known property of linear algebra. This unique generic condition is identical to that provided in previous works. Application of this method to the famous coupled FitzHugh-Nagumo equations and to the Hodgkin-Huxley model enables showing the existence of "canard solutions" in such systems.

1. Introduction

In the beginning of the eighties, Benoît and Lobry [1], Benoît [2], and then Benoît [3] in his PhD- thesis studied canard solutions in $\mathbb{R}^3$. In the article entitled "Systèmes Lents-Rapides dans $\mathbb{R}^3$ et Leurs Canards," Benoît [2, p. 170] proved the existence of canards solution for three-dimensional singularly perturbed systems with two *slow* variables and one *fast* variable while using "nonstandard analysis" according to a theorem which stated that canard solutions exist in such systems provided that the *pseudo singular point* (this concept has been originally introduced by Argémi [4]; see Section 2.8) of the *slow dynamics*, that is, of the *reduced vector field,* is of *saddle*-type. Nearly twenty years later, Szmolyan and Wechselberger [5] extended "Geometric Singular Perturbation Theory (see Fenichel [6, 7], O'Malley [8], Jones [9], and Kaper [10])" to canards problems in $\mathbb{R}^3$ and provided a "standard version" of Benoît's theorem [2]. Very recently, Wechselberger [11] generalized this theorem for n-dimensional singularly perturbed systems with k *slow* variables and m *fast* (1). The methods used by Szmolyan and Wechselberger [5] and Wechselberger [11] require implementing a "desingularization procedure" which can be summarized as follows: first, they compute the *normal form* of such singularly perturbed systems which is expressed according to some coefficients (a and b for dimension three and $\tilde{a}$, $\tilde{b}$, and $\tilde{c}_j$ for dimension four) depending on the functions defining the original vector field and their partial derivatives with respect to the variables. Secondly, they project the "desingularized vector field" (originally called "normalized slow dynamics" by Benoît [2, p. 166]) of such a *normal form* on the tangent bundle of the critical manifold. Finally, they evaluate the Jacobian of the projection of this "desingularized vector field" at the *folded singularity* (originally called *pseudo singular points* by Argémi [4, p. 336]). This leads Szmolyan and Wechselberger [5, p. 427] and Wechselberger [11, p. 3298] to a "classification of *folded singularities* (*pseudo singular points*)." Thus, they showed that for three-dimensional singularly perturbed systems such *folded singularity* is of *saddl-type* if the following condition is satisfied, $a < 0$ while for four-dimensional singularly perturbed systems such *folded singularity* is of *saddle-type* if $\tilde{a} < 0$. Then, Szmolyan and Wechselberger [5, p. 439] and Wechselberger [11, p. 3304] established their Theorem 4.1. which states that "In the folded saddle and in the folded node case singular