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On the integrability of Liénard systems with a strong saddle

Jaume Giné[a,*], Jaume Llibre[b]

[a] *Departament de Matemàtica, Inspires Research Centre, Universitat de Lleida, Avda. Jaume II, 69; 25001 Lleida, Catalonia, Spain*
[b] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*

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ABSTRACT

We study the local analytic integrability for real Liénard systems, $\dot{x} = y - F(x)$, $\dot{y} = x$, with $F(0) = 0$ but $F'(0) \neq 0$, which implies that it has a strong saddle at the origin. First we prove that this problem is equivalent to study the local analytic integrability of the $[p : -q]$ resonant saddles. This result implies that the local analytic integrability of a strong saddle is a hard problem and only partial results can be obtained. Nevertheless this equivalence gives a new method to compute the so-called resonant saddle quantities transforming the $[p : -q]$ resonant saddle into a strong saddle.

1. Introduction and main results

Two of the main problems about the planar Liénard differential systems are to know whether they are integrable or not, and to give criteria for controlling their number of limit cycles. In this work we deal with the first problem. For more information on the second mentioned problem, see [1]. Liénard differential systems appeared in electrical circuits with nonlinear elements (see [2,3]), but later many other situations have been modeled by these type of differential equations, see [4–6] and references therein.

Assume that a real planar analytic differential system has a weak focus. It is well-known that this singular point is a center if and only if the equation has an analytic first integral defined in a neighborhood of this point, see for instance [7]. Consequently the center problem for non-degenerate singular points is equivalent to the local analytic integrable problem for such singular points.

The following theorem characterizes the centers for the classical real analytic Liénard systems, see the proof in [8–10].

* Corresponding author.
E-mail addresses: gine@matematica.udl.cat (J. Giné), jllibre@mat.uab.cat (J. Llibre).

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