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ON THE MECHANISMS FOR PRODUCING LINEAR TYPE CENTERS IN POLYNOMIAL DIFFERENTIAL SYSTEMS

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Abstract. In this paper we study the mechanisms that provide linear type centers in the polynomial differential systems. These mechanisms are the algebraic reducibility and the Liouvillian integrability. We state a conjecture and some open questions. These mechanisms also explain all the known nilpotent and degenerate centers.

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1. Introduction and Preliminary Results

A *center* for a real analytic differential system in the plane

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

is a singularity p for which there exist a neighborhood U such that $U \setminus \{p\}$ is filled with periodic orbits.

A center is of *linear type* if the eigenvalues of its linear part are purely imaginary. Determine linear type centers is a classical problem in the qualitative theory of differential equations, see for instance [21], [31], [32], [39], [41]. The linear type centers of the analytic differerential systems (1) are characterized by a theorem of Poincaré–Liapunov [32], [39], which says that a center is of linear type if and only if the system has a non-constant analytic first integral defined in a neighborhood of it.

We say that the differential system (1) is *polynomial* when the functions P and Q are polynomials. The *degree* of a polynomial differential system (1) is the maximum of the degrees of the polynomials P and Q.

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