International Journal of Bifurcation and Chaos, Vol. 23, No. 4 (2013) 1330010 (13 pages)

DOI: 10.1142/S0218127413300103

# CANARDS FROM CHUA'S CIRCUIT

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Received January 16, 2013

The aim of this work is to extend Benoît's theorem for the generic existence of "canards" solutions in *singularly perturbed dynamical systems* of dimension three with one fast variable to those of dimension four. Then, it is established that this result can be found according to the *Flow Curvature Method*. Applications to Chua's cubic model of dimension three and four enable to state the existence of "canards" solutions in such systems.

*Keywords*: Geometric singular perturbation method; flow curvature method; singularly perturbed dynamical systems; canard solutions.



## 1. Introduction

Many systems in biology, neurophysiology, chemistry, meteorology, electronics exhibit several time scales in their evolution. Such systems, today called *singularly perturbed dynamical systems*, have been modeled by a system of differential equations (1) having a small parameter multiplying one or several components of its vector field. Since the works of Andronov and Khaikin [1937], Tikhonov [1948], the *singular perturbation method*[1] has been the subject of many research works, among which we will quote those of Argémi [1978] who carefully studied the slow motion. According to Tikhonov [1948], Takens [1976], Jones [1994] and Kaper [1999], *singularly perturbed systems* may be defined as:

$$\mathbf{x}' = \varepsilon \mathbf{f}(\mathbf{x}, \mathbf{y}, \varepsilon), \quad \mathbf{y}' = \mathbf{g}(\mathbf{x}, \mathbf{y}, \varepsilon), \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^p$, $\mathbf{y} \in \mathbb{R}^m$, $\varepsilon \in \mathbb{R}^+$, and the prime denotes differentiation with respect to the independent variable $t$. The functions $\mathbf{f}$ and $\mathbf{g}$ are assumed to be $C^\infty$ functions[2] of $\mathbf{x}$, $\mathbf{y}$ and $\varepsilon$ in $U \times I$, where $U$ is an open subset of $\mathbb{R}^p \times \mathbb{R}^m$ and $I$ is an open interval containing $\varepsilon = 0$.

[1]For an introduction to singular perturbation method see [O'Malley, 1974; Kaper, 1999].
[2]In certain applications these functions are supposed to be $C^r$, $r \geq 1$.