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CANARDS EXISTENCE IN THE HINDMARSH–ROSE MODEL

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Abstract. In two previous papers we have proposed a new method for proving the existence of "canard solutions" on one hand for three and four-dimensional singularly perturbed systems with only one *fast* variable and, on the other hand for four-dimensional singularly perturbed systems with two *fast* variables [J.M. Ginoux and J. Llibre, *Qual. Theory Dyn. Syst.* **15** (2016) 381–431; J.M. Ginoux and J. Llibre, *Qual. Theory Dyn. Syst.* **15** (2015) 342010]. The aim of this work is to extend this method which improves the classical ones used till now to the case of three-dimensional singularly perturbed systems with two *fast* variables. This method enables to state a unique generic condition for the existence of "canard solutions" for such three-dimensional singularly perturbed systems which is based on the stability of *folded singularities* (*pseudo singular points* in this case) of the *normalized slow dynamics* deduced from a well-known property of linear algebra. Applications of this method to a famous neuronal bursting model enables to show the existence of "canard solutions" in the Hindmarsh-Rose model.

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1. Introduction

The concept of "canard solutions" for three-dimensional singularly perturbed systems with two *slow* variables and one *fast* has been introduced in the beginning of the eighties by Benoît and Lobry [5], Benoît [4]. Their existence has been proved by Benoît ([4], p. 170) in the framework of "Non-Standard Analysis" according to a theorem which states that canard solutions exist in such systems provided that the *pseudo singular point* of the *slow dynamics*, *i.e.*, of the *reduced vector field* is of *saddle* type. Nearly twenty years later, while using the so-called "blow-up" technique they introduced, Dumortier and Roussarie [6] and then, Szmolyan and Wechselberger [31] provided a "standard version" of Benoît's theorem [4]. Recently, Wechselberger [37] generalized this theorem for n-dimensional singularly perturbed systems with k *slow* variables and m *fast* (where $n = k + m$). The method they used require to implement a "desingularization procedure" which can be summarized as follows: first, they compute the *normal form* of such singularly perturbed systems which is expressed according to some coefficients (a and b for dimension three and $\tilde{a}$, $\tilde{b}$ and $\tilde{c}_1$ for

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