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Centers for a class of generalized quintic polynomial differential systems

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ABSTRACT

We characterize the centers of the planar polynomial differential systems of degree $d \geqslant 5$ odd that in complex notation writes as

$$\dot{z} = iz + (z\bar{z})^{\frac{d-5}{2}}(Az^5 + Bz^4\bar{z} + Cz^3\bar{z}^2 + Dz^2\bar{z}^3 + Ez\bar{z}^4 + F\bar{z}^5),$$

where $A, B, C, D, E, F \in \mathbb{C}$ and either $A = \mathrm{Re}(E) = 0$, or $A = \mathrm{Im}(E) = 0$, or $\mathrm{Re}(A) = E = 0$, or $\mathrm{Im}(A) = E = 0$.

1. Introduction and statement of the main results

The *center-focus problem*, i.e. the problem of distinguishing between a center and a focus, is one of the main problems in the qualitative theory of real planar polynomial systems. For singular points whose linear differential part have eigenvalues purely imaginary this problem is equivalent to the existence of an analytic first integral in a neighborhood of the singular point, see for more details the references [2,11].

As usual a singular point is a *focus* if it has neighborhood such that all the orbits spiral either in forward or in backward time to the singular point, and a singular point is a *center* if it has a neighborhood such that all the orbits in this neighborhood are periodic except the singular point.

In this paper we study the center-focus problem for a class of polynomial differential systems which generalizes the class of quintic polynomial differential systems with homogeneous nonlinearities. The classification of the centers for different classes of polynomial differential systems begun with the quadratic polynomial differential systems and the class of cubic polynomial differential systems with only homogeneous nonlinearities, see [1,23–26]. The complete classification of the centers for the class of all polynomial differential systems of degree 3 is still open. The centers of polynomial differential systems of the form a linear center plus a homogeneous polynomial of degree $k > 3$ are not classified, but there are some results for $k = 4, 5, 6, 7, 9$ see [3,4,10,18–21]. The main problem for ending these classifications is the huge amount of computations which usually for computing the so called Lyapunov constants, see for more details [15] and references therein.

In this paper we study the center-focus problem for the real polynomial differential systems in the plane that has a singular point at the origin with eigenvalues $\pm i$ and that can be written in complex notation as

$$\dot{z} = iz + (z\bar{z})^{\frac{d-5}{2}}(Az^5 + Bz^4\bar{z} + Cz^3\bar{z}^2 + Dz^2\bar{z}^3 + Ez\bar{z}^4 + F\bar{z}^5), \tag{1}$$

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