Journal of Computational and Applied Mathematics 279 (2015) 173–186

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Centers and isochronous centers for generalized quintic systems

Jaume Giné [a,*], Jaume Llibre [b], Claudia Valls [c]

[a] Departament de Matemàtica, Universitat de Lleida, Avda. Jaume II, 69, 25001 Lleida, Catalonia, Spain

[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

[c] Departamento de Matemática, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001, Lisboa, Portugal

ARTICLE INFO

Article history:
Received 18 December 2013
Received in revised form 6 November 2014

MSC:
primary 34C05
secondary 37C10

Keywords:
Non-degenerate center
Poincaré–Liapunov–Abel constants
Gröbner basis theory
Computation on modular arithmetics

ABSTRACT

In this paper we classify the centers and the isochronous centers of certain polynomial differential systems in $\mathbb{R}^2$ of degree $d \geq 5$ odd that in complex notation are

$$\dot{z} = (\lambda + i)z + (z\bar{z})^{\frac{d-5}{2}}(Az^5 + Bz^4\bar{z} + Cz^3\bar{z}^2 + Dz^2\bar{z}^3 + Ez\bar{z}^4 + F\bar{z}^5),$$

where $z = x + iy$, $\lambda \in \mathbb{R}$ and $A, B, C, D, E, F \in \mathbb{C}$. Note that if $d = 5$ we obtain the class of polynomial differential systems in the form of a linear system with homogeneous polynomial nonlinearities of degree 5.

Due to the huge computations required for computing the necessary and sufficient conditions for the characterization of the centers and isochronous centers, our study uses algorithms of computational algebra based on the Gröbner basis theory and on modular arithmetics.

1. Introduction and statement of the main results

The center–focus problem is one of the main problems in the qualitative theory of real planar polynomial systems. For nondegenerate singular points this problem is equivalent to have an analytic first integral in a neighborhood of the singular point, see [1–5].

We recall that a singular point is a *center* if it has a neighborhood such that all the orbits, with the exception of the singular point, in this neighborhood are periodic, and that a singular point is a *focus* if it has neighborhood such that all the orbits, with the exception of the singular point, spiral either in forward or in backward time to the singular point.

In this paper we study the center–focus problem for a class of polynomial systems which generalize the class of polynomial systems with homogeneous nonlinearities of degree 5. The characterization of the centers of the polynomial differential systems begun with the quadratic ones and the class of cubic polynomial systems with only homogeneous nonlinearities, see [6–9]. See [10,11] for an update on these cases. Actually we are very far from obtaining a complete classification of the centers for the class of all polynomial systems of degree 3. However some subclasses of cubic systems with centers are known, see for instance [12,13] and references therein. The centers of polynomial systems in the form of a linear center with homogeneous polynomial nonlinearities of degree $k > 3$ are not classified, but there are partial results for $k = 4, 5, 6, 7$ see [14–25]. However the huge amount of computations which usually are necessary becomes the center problem in general computationally intractable, see [26] and references therein.

* Corresponding author.
E-mail addresses: gine@matematica.udl.cat (J. Giné), jllibre@mat.uab.cat (J. Llibre), cvalls@math.ist.utl.pt (C. Valls).

http://dx.doi.org/10.1016/j.cam.2014.11.007
0377-0427/