# THE CUBIC POLYNOMIAL DIFFERENTIAL SYSTEMS WITH TWO CIRCLES AS ALGEBRAIC LIMIT CYCLES

JAUME GINÉ[1], JAUME LLIBRE[2] AND CLAUDIA VALLS[3]

ABSTRACT. In this paper we characterize all cubic polynomial differential systems in the plane having two circles as invariant algebraic limit cycles.

[1]

## 1. Introduction and statement of the main results

A *planar polynomial differential system* is a differential system of the form

$$\begin{aligned} \dot{x} &= P(x,y), \\ \dot{y} &= Q(x,y), \end{aligned} \tag{1}$$

where $P$ and $Q$ are real polynomials. We say that the polynomial differential system (1) has *degree* $n$, if $n$ is the maximum of the degrees of the polynomials $P$ and $Q$. Usually a *polynomial differential system of degree* 3 is denoted simply as a *cubic system*. The dot in (1) denotes derivative with respect to the independent variable $t$.

In this paper we want to analyze all cubic polynomial differential systems having two circles as algebraic limit cycles.

In [4] the authors proved, first that every planar polynomial vector field of degree $n$ with $n$ invariant circles is Darboux integrable without limit cycles, and second that a planar polynomial vector field of degree $n$ has at most $n-1$ invariant circles as algebraic limit cycles. So, in particular, cubic systems have at most two circles as algebraic limit cycles.

Our first result is to provide a normal form for all cubic polynomial differential systems having two circles as invariant algebraic curves.

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[1]The corresponding author is Jaume Llibre.