CENTERS OF WEIGHT-HOMOGENEOUS POLYNOMIAL VECTOR FIELDS ON THE PLANE

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Abstract. We characterize all centers of a planar weight-homogeneous polynomial vector fields. Moreover we classify all centers of a planar weight-homogeneous polynomial vector fields of degrees 6 and 7.

1. Introduction and statement of the main results

One of the main problems in the qualitative theory of real planar polynomial differential systems is the center-focus problem. This problem consists in distinguish when a singular point is either a focus or a center. The notion of center and focus goes back to Poincaré [18]. A singular point p of system (1) is a *center* if there is a neighborhood of p fulfilled of periodic orbits with the unique exception of p. The *period annulus* of a center is the region fulfilled by all the periodic orbits surrounding the center. We say that a center located at the origin is *global* if its period annulus is $\mathbb{R}^2 \setminus \{(0,0)\}$.

The center problem for planar polynomial vector fields has been intensively studied. The center problem for linear type singular points, i.e., singular points with imaginary pure eigenvalues is the most studied. It started with the study of the quadratic polynomial differential systems with linear type singular points. The works of Dulac [5], Bautin [4], Żoładek [21], are the principal ones for the quadratic case, see Schlomiuk [20] for an update of this works. But the center-focus problem for polynomial differential systems of degree larger than two remains open. However, for polynomial differential systems of degree larger than two, there are richer partial results on the center problem, see for instance [8, 19, 22, 23].

The inability to go beyond in the study of centers for general polynomial differential systems has motivated the study of particular cases as they are the quasi-homogeneous or weight-homogeneous polynomial differential systems.

Hence we consider the polynomial differential systems of the form

$$\dot{x} = P(x,y), \quad \dot{y} = Q(x,y), \tag{1}$$

where $P, Q \in \mathbb{R}[x,y]$ are coprime and the origin is a singularity of system (1). As usual, $\mathbb{R}[x,y]$ denotes the ring of polynomials in the variables x and y with coefficients in $\mathbb{R}$, and the dot denotes derivative with respect to an independent variable t.

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