Journal of Applied Analysis and Computation
Volume 7, Number 4, November 2017, 1534–1548
Website:http://jaac-online.com/
DOI:10.11948/2017093

CENTERS FOR THE KUKLES HOMOGENEOUS SYSTEMS WITH EVEN DEGREE

Jaume Giné[1,†], Jaume Llibre[2] and Claudia Valls[3]

Abstract For the polynomial differential system $\dot{x} = -y$, $\dot{y} = x + Q_n(x, y)$, where $Q_n(x, y)$ is a homogeneous polynomial of degree n there are the following two conjectures done in 1999. (1) Is it true that the previous system for $n \geq 2$ has a center at the origin if and only if its vector field is symmetric about one of the coordinate axes? (2) Is it true that the origin is an isochronous center of the previous system with the exception of the linear center only if the system has even degree? We give a step forward in the direction of proving both conjectures for all n even. More precisely, we prove both conjectures in the case $n = 4$ and for $n \geq 6$ even under the assumption that if the system has a center or an isochronous center at the origin, then it is symmetric with respect to one of the coordinate axes, or it has a local analytic first integral which is continuous in the parameters of the system in a neighborhood of zero in the parameters space. The case of n odd was studied in [8].

Keywords Center-focus problem, isochronous center, Poincaré-Liapunov constants, Gröbner basis of polynomial systems.

MSC(2010) 34C05, 37C10.

1. Introduction and statement of the main results

The conditions under which the origin for the differential system of the form

$$\dot{x} = -y, \quad \dot{y} = x + a_1x^2 + a_2xy + a_3y^2 + a_4x^3 + a_5x^2y + a_6xy^2 + a_7y^3, \tag{1.1}$$

is a center were found in [11]. During may years it had been thought that these conditions were necessary and sufficient conditions, but some new centers have been found later on, see [2,10]. In [4] the center problem for the class of system (1.1) with $a_7 = 0$ was solved, and it was proved that at most five limit cycles bifurcate from the origin. In [12] it was solved the center problem for system (1.1) when $a_2 = 0$ and it was proved that at most six limit cycles bifurcate from the origin. The first complete solution of the center-focus problem of system (1.1) was obtained in [13]. Using the Cherkas' method of passing to a Liénard equation, in [16] it was also given the complete solution of the center-focus problem of system (1.1), see also the

†the corresponding author. Email address: gine@matematica.udl.cat (J. Giné)
[1]Departament de Matemàtica, Universitat de Lleida, Avda. Jaume II, 69; 25001 Lleida, Catalonia, Spain
[2]Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
[3]Departamento de Matemática, Instituto Superior Técnico, Av. Rovisco Pais 1049-001, Lisboa, Portugal