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CENTERS FOR GENERALIZED QUINTIC POLYNOMIAL DIFFERENTIAL SYSTEMS

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ABSTRACT. We classify the centers of polynomial differential systems in $\mathbb{R}^2$ of odd degree $d \geq 5$, in complex notation, as $\dot{z} = iz + (z\overline{z})^{(d-5)/2}(Az^5 + Bz^4\overline{z} + Cz^3\overline{z}^2 + Dz^2\overline{z}^3 + Ez\overline{z}^4 + F\overline{z}^5)$, where $A, B, C, D, E, F \in \mathbb{C}$ and either $A = \mathrm{Re}(D) = 0$, $A = \mathrm{Im}(D) = 0$, $\mathrm{Re}(A) = D = 0$ or $\mathrm{Im}(A) = D = 0$.

1. Introduction and statement of the main results. In the qualitative theory of real planar polynomial differential systems one of the main problems is the *center-focus problem*, i.e., the problem of distinguishing between a center and a focus. For singular points whose linear part has a pair of pure imaginary eigenvalues, this problem is equivalent to the existence of an analytic first integral defined in a neighborhood of the singular point, see, for more details, [**2, 12, 13, 24, 25**].

A singular point is a *center* if there exists a neighborhood of it such that all of the orbits in this neighborhood are periodic except the singular point, and a singular point is a *focus* if there is a neighborhood of it such that all of the orbits in this neighborhood spiral either in forward or in backward time to the singular point.

We study the center-focus problem for a class of polynomial differential systems which generalize the class of linear polynomial differential

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