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Simultaneity of centres in $\mathbb{Z}_q$-equivariant systems

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We study the simultaneous existence of centres for two families of planar $\mathbb{Z}_q$-equivariant systems. First, we give a short review about $\mathbb{Z}_q$-equivariant systems. Next, we present the necessary and sufficient conditions for the simultaneous existence of centres for a $\mathbb{Z}_2$-equivariant cubic system and for a $\mathbb{Z}_2$-equivariant quintic system.

1. Introduction

The second part of Hilbert's 16th problem deals with the existence of a uniform upper bound on the number of limit cycles $H(n)$ of a planar polynomial differential system

$$\dot{x} = P(x, y) \quad \text{and} \quad \dot{y} = Q(x, y), \tag{1.1}$$

in function of its degree n, where $n = \max(\deg P, \deg Q)$ (see, for instance, [1,2]). It is well known that linear polynomial differential systems have no limit cycles, so $H(1) = 0$. For $n \geq 2$, the problem remains open. Only lower bounds for $H(n)$ with $n \geq 2$ are known and our objective is to improve these lower bounds. An efficient method to do this is to perturb symmetric Hamiltonian systems. Symmetric Hamiltonian systems are Hamiltonian systems with certain symmetries that allow the existence of a great number of centres whose perturbations can produce a large number of limit cycles.

A generalization of these symmetric Hamiltonian systems are the $\mathbb{Z}_q$-equivariant systems defined below using a cyclic group $\mathbb{Z}_q$. First, we give a survey on the existing results related to the local and global bifurcations of limit cycles for such systems perturbing their centres. The existence of a $\mathbb{Z}_q$-symmetry implies that, in analysing the number of limit cycles which

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