CHIELLINI HAMILTONIAN LIÉNARD DIFFERENTIAL SYSTEMS

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Abstract. We characterize the centers of the Chiellini Hamiltonian Liénard second-order differential equations $x' = y$, $y' = -f(x)y - g(x)$ where $g(x) = f(x)(k - \alpha(1+\alpha)\int f(x)dx)$ with $\alpha, k \in \mathbb{R}$. Moreover we study the phase portraits in the Poincaré disk of these systems when $f(x)$ is linear.

1. Introduction

In the study of differential equations, a Liénard equation in $\mathbb{R}$ is a second–order differential equation of the form

$$x'' + f(x)x' + g(x) = 0,$$

named in honor of the French physicist Alfred-Marie Liénard [13], who during the development of radio and vacuum tube technology, introduced such equations to model oscillating circuits. These equations have been studied intensively, thus now in MathSciNet are more than 1480 articles which appear the keywords "Liénard" and "equation", some of these recent papers are for instance [2, 6, 7, 10, 12, 14] and the references quoted therein.

Here we deal with Liénard second–order differential equations

$$x'' + f(x)x' + g(x) = 0,$$

where f and g are polynomials, or equivalently with first–order differential system of equations

$$\dot{x} = y, \qquad \dot{y} = -f(x)y - g(x), \tag{1}$$

in $\mathbb{R}^2$. In 1931 Chiellini [3] proved that system (1) is integrable if the functions $f(x)$ and $g(x)$ satisfy the condition

$$\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right) = s f(x), \tag{2}$$

where s is a constant. This condition is now known as the Chiellini condition.

Recently Ghose Choudhury and Guha in [9] studied when the Liénard systems (1) satisfying Chiellini condition admit a Hamiltonian formulation

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