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A COLLOCATION METHOD FOR THE NUMERICAL FOURIER ANALYSIS OF QUASI–PERIODIC FUNCTIONS. II: ANALYTICAL ERROR ESTIMATES

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Abstract. In a previous paper [6], a numerical procedure for the Fourier analysis of quasi–periodic functions was developed, allowing for an accurate determination of frequencies and amplitudes from equally–spaced samples of the input function on a finite time interval. This paper is devoted to a complete error analysis of that procedure, from which computable bounds are deduced. These bounds are verified and further discussed in examples.

1. **Introduction.** In a previous paper [6] a numerical procedure has been developed to solve the following frequency analysis problem: given the samples

$$f(jT/N), \qquad j = 0, ..., N-1,$$

of a real-valued function $f(t)$, equally spaced on an interval $[0,T]$, determine a trigonometric polynomial,

$$Q_f(t) = A_0^c + \sum_{l=1}^{N_f} \big(A_l^c \cos(2\pi\nu_l t/T) + A_l^s \sin(2\pi\nu_l t/T) \big), \tag{1}$$

whose frequencies, $\{\nu_l\}_{l=1}^{N_f}$, and amplitudes, $\{A_l^c\}_{l=0}^{N_f}$, $\{A_l^s\}_{l=1}^{N_f}$, are a good approximation of the corresponding ones of $f(t)$. The number of frequencies, N_f, is to be determined by the procedure (in terms of some input parameters), and we want it to be as small as possible, while keeping high accuracy in the frequencies and amplitudes computed.

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