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A COLLOCATION METHOD FOR THE NUMERICAL FOURIER ANALYSIS OF QUASI–PERIODIC FUNCTIONS. I: NUMERICAL TESTS AND EXAMPLES

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Abstract. The purpose of this paper is to develop a numerical procedure for the determination of frequencies and amplitudes of a quasi–periodic function, starting from equally–spaced samples of it on a finite time interval. It is based on a collocation method in frequency domain. Strategies for the choice of the collocation harmonics are discussed, in order to ensure good conditioning of the resulting system of equations. The accuracy and robustness of the procedure is checked with several examples. The paper is ended with two applications of its use as a dynamical indicator. The theoretical support for the method presented here is given in a companion paper [21].

1. Introduction.

The goal of this paper is the development of a numerical method to solve the following frequency analysis problem: given N samples $\{f(jT/N)\}_{j=0}^{N-1}$ of a real-valued function $f(t)$, equally spaced on the interval $[0,T]$, determine a trigonometric polynomial,

$$Q_f(t) = A_0^c + \sum_{l=1}^{N_f}\big(A_l^c\cos(2\pi\nu_l t/T) + A_l^s\sin(2\pi\nu_l t/T)\big), \tag{1}$$

whose frequencies $\{\nu_l\}_{l=1}^{N_f}$, and amplitudes, $\{A_l^c\}_{l=0}^{N_f}$, $\{A_l^s\}_{l=1}^{N_f}$, are a good approximation of the ones of $f(t)$. The number of frequencies, N_f, has to be determined (in terms of some input parameters), and we want it to be as small as possible while keeping high accuracy in the frequencies and amplitudes computed.

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