## A Survey on the Set of Periods of the Graph Homeomorphisms

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**Abstract** In this paper we summarize the known results on the possible sets of periods of homeomorphisms defined on some classes of finite connected compact graphs, and we present new results.

Keywords Homeomorphisms · Topological graph · Periods · Periodic points

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## 1 Introduction

Here a (*topological graph*) or simply a *graph* G is a compact set formed by a finite union of *vertices* (points) and *edges*, which are homeomorphic to a non-empty open interval of the real line, and are pairwise disjoint. The boundary of one edge is formed either by two vertices, or by a unique vertex. Moreover, the graphs that we consider here always are connected.

We identify a circle with the unit circle  $S^1$  centered at the origin of the complex plane. A *circuit* (or *loop*) of a graph *G* is any subset of *G* homeomorphic to  $S^1$ . A *tree* is a graph without circuits. The set of vertices of a graph *G* will be denoted by V(G). Clearly V(G) is finite.

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