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On limit cycles bifurcating from the infinity in discontinuous piecewise linear differential systems

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ABSTRACT

In this paper we consider the linear differential center $(\dot{x}, \dot{y}) = (-y, x)$ perturbed inside the class of all discontinuous piecewise linear differential systems with two zones separated by the straight line $y = 0$. Using the Bendixson transformation we provide sufficient conditions to ensure the existence of a crossing limit cycle coming purely from the infinity. We also study the displacement function for a class of discontinuous piecewise smooth differential system.

1. Introduction and statement of the main result

Establishing sufficient conditions for the existence of periodic solutions, specially *limit cycles*, i.e. periodic solutions isolated in the set of all periodic solutions, is one of the main problems of the qualitative theory of planar differential systems. A classical way to produce and to study limit cycles is perturbing the periodic solutions of a *center*, which is a point having a neighborhood, except itself, filled by periodic solutions. This problem for smooth differential systems in the plane has been studied intensively, either for limit cycles bifurcating from finite periodic solutions (see, for instance, hundreds of references in the book [4]), or for limit cycles bifurcating from the infinity (see, for instance, [7,21,22]). In [8] the authors studied the bifurcation of a limit cycle from the infinity for a non-smooth but continuous piecewise differential system. Nevertheless, as far as we know, there are no studies of limit cycles bifurcating from the infinity for discontinuous differential systems. Accordingly, this is the objective of the present paper: to study limit cycles coming from the infinity for a class of discontinuous differential systems.

Recently the theory of discontinuous differential systems has been strongly developed, with growing importance at the frontier between mathematics, physics, engineering, and the life sciences. Interest stems particularly from discontinuous dynamical models (see, for instance, [15,20]). Additionally, the existence of periodic solutions gives important information about the dynamics of these models, as the presence of oscillatory motions (see, for instance, the book of Andronov, Vitt, and Khaikin [2], and the book of Minorski [16], both classical and important works on physical oscillatory phenomena), which represents one of the main source of physical motivation for this study. There are some works dealing with limit cycles bifurcating from finite periodic solutions in discontinuous differential systems, see, for instance, [3,9,10,12–14,17,18]. To the best of our knowledge, this paper is the first work dealing with limit cycles bifurcating from the infinity for discontinuous systems.

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