Applied Mathematics Letters 103 (2020) 106189

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

24 crossing limit cycles in only one nest for piecewise cubic systems

Luiz F.S. Gouveia, Joan Torregrosa *

Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

ARTICLE INFO

Article history:
Received 8 October 2019
Received in revised form 10 December 2019
Accepted 10 December 2019
Available online 18 December 2019

Keywords:
Cubic piecewise vector field
Piecewise center cyclicity
Lyapunov quantities

ABSTRACT

In this work, we are interested in crossing limit cycles surrounding only one equilibrium point or a sliding segment. The studied systems are piecewise cubic polynomial defined in two zones separated by a straight line. In this class, we get at least 24 crossing limit cycles, all of them in only one nest, bifurcating from a cubic polynomial center. The computations use a parallelization algorithm.

1. Introduction

Andronov, in [1], started the study of piecewise linear systems. In last years, this subject has been widely studied, since many problems of engineering, physics, and biology can be modeled by such systems, see [2,3]. Most of usual models propose piecewise differential systems defined in two half planes separated by a straight line. Recently, the study of the number of limit cycles have received a special attention, see for example [4–6]. In particular, it can be considered as a generalization of the 16th-Hilbert problem, see more details on this classical problem in [7].

In this paper, we are interested in the study of isolated periodic orbits, the so-called limit cycles, for piecewise differential equations of the form

$$\begin{cases} (x', y') = (P^+(x,y,\lambda), Q^+(x,y,\lambda)), \text{ when } y > 0, \\ (x', y') = (P^-(x,y,\lambda), Q^-(x,y,\lambda)), \text{ when } y < 0, \end{cases} \tag{1}$$

where $P^{\pm}(x,y,\lambda)$ and $Q^{\pm}(x,y,\lambda)$ are polynomials of degree n in (x,y) and $\lambda \in \mathbb{R}^K$, where K is the total number of parameters. It is not restrictive to consider the straight line $\Sigma = \{y = 0\}$, it divides the real plane in two half-planes $\Sigma^{\pm} = \{(x,y) : \pm y > 0\}$, and the trajectories on Σ are defined following the Filippov convention, see [3]. We will consider only limit cycles of crossing type, that is, when both vector fields point out in the same direction in the intersection points with the discontinuity line Σ. We can define

* Corresponding author.
E-mail addresses: fernandosg@mat.uab.cat (L.F.S. Gouveia), torre@mat.uab.cat (J. Torregrosa).

https://doi.org/10.1016/j.aml.2019.106189