HOUSTON JOURNAL OF MATHEMATICS

Volume 23, No. 3, 1997

ORDERS AND PERIODS OF ALGEBRAICALLY-FINITE SURFACE MAPS

JOHN GUASCHI AND JAUME LLIBRE
COMMUNICATED BY JAMES DAMON

ABSTRACT. Let M be a compact, connected, orientable surface of genus g without boundary, and let $f\colon M \to M$ be a continuous map such that all the eigenvalues of the induced action on rational homology are roots of unity. We present an algorithm to compute as a function of g: firstly, all the algebraic orders; secondly, the potential periods; and thirdly, the potential least periods. We apply the algorithm up to genus 5 for degree -1 and up to genus 3 for degree $+1$. Also, we give results on the algebraic orders and potential periods for arbitrary genus. For instance, if $g > 2$ then the map has a period less than or equal to $2g-2$. Moreover, for degree -1, the algebraic orders are always even, and if $g > 1$ is even then the set of algebraic orders for the surfaces of genus g and $g+1$ are equal. We improve some of these results in the particular case that f is a finite order homeomorphism.

1. INTRODUCTION AND STATEMENT OF MAIN RESULTS

In dynamical systems it is often the case that topological information can be used to study qualitative and quantitative properties of the system. This paper deals with the elucidation of the set of periods of surface maps of degree ± 1 through their induced action on homology. Let $f\colon M \to M$ be a continuous map of a surface M. A point $x \in M$ is called *periodic* if there exists some $n \in \mathbb{N}$ such that $f^n(x) = x$. We call the least such n the *period* of x under f. The set of periods of all periodic points of f is denoted by $\mathrm{Per}(f)$.

Let M be a compact, connected, orientable surface of genus g without boundary, or more simply, a *surface*. If $f\colon M \to M$ is a continuous map of degree ± 1 then we shall call it a *surface map*. Such a map is always homotopic to a homeomorphism [4]. Moreover, if a surface map f is such that the eigenvalues of

1991 *Mathematics Subject Classification.* Primary 58F20.
Key words and phrases. periods, orders, surface maps.