

On the pairs of points of the Kolyada's triangular map.

Juan Luis García Guirao*

Abstract

The aim of the present paper is to describe completely the dynamics of a triangular map F introduced by S. Kolyada in [Ergod. Theor. Dyn. Syst. 12, 749-768, 1992] complementing the results given in [F. Balibrea *et al.*, Far East J. Dyn. Sys. 3(1), 87-101, 2001]. We prove on one hand that there is no approximately periodic points which are not fixed and on other hand that each pair of distinct points $\{x, y\} \in I^2 \setminus I_0$ is an asymptotic pair under the iteration of F .

1 Introduction and Notation

Let X be a compact metric space with metric ϱ and $\psi : X \rightarrow X$ be a continuous self-map. The pair (X, ψ) is called the *discrete dynamical system* generated by ψ on X . For every point $x \in X$ its *orbit* by ψ is the set $\{\psi^n(x)\}_{n=0}^\infty$ where $\psi^n(x) = \psi(\psi^{n-1}(x))$ for $n \geq 1$ and ψ^0 is the identity map on X . A point $y \in X$ belongs to the ω -*limit set* of $x \in X$ by ψ , denoted by $\omega_\psi(x)$, if there exists a sequence of positive integers $(n_k)_{k=0}^\infty$ such that $\psi^{n_k}(x) \rightarrow y$ where $n_k \rightarrow \infty$. A pair of points $\{x, y\} \subseteq X$ is said to be a *Li-Yorke pair* if one has simultaneously

*This paper has been partially supported by MCYT (Ministerio de Ciencia y Tecnología, Spain) and FEDER (Fondo Europeo de Desarrollo Regional), grants BEC2001-0535 and BFM2002-03512; Fundación Séneca (Comunidad Autónoma de la Región de Murcia), grant PI-8-00807-FS-01 and JCCM (Junta de Comunidades de Castilla-La Mancha), grant PAC-02-002.

Mathematics Subject Classification 2000: 37B20, 37B99..

Keywords and phrases: discrete dynamical system, triangular map, asymptotic pair, approximately periodic point, ω -limit set.