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PERIODS FOR TRANSVERSAL MAPS VIA LEFSCHETZ NUMBERS FOR PERIODIC POINTS

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ABSTRACT. Let $f: M \to M$ be a C^1 map on a C^1 differentiable manifold. The map f is called transversal if for all $m \in \mathbb{N}$ the graph of f^m intersects transversally the diagonal of $M \times M$ at each point (x, x) such that x is a fixed point of f^m. We study the set of periods of f by using the Lefschetz numbers for periodic points. We focus our study on transversal maps defined on compact manifolds such that their rational homology is $H_0 \approx \mathbb{Q}$, $H_1 \approx \mathbb{Q} \oplus \mathbb{Q}$ and $H_k \approx \{0\}$ for $k \neq 0, 1$.

1. Introduction and statement of the main results

In dynamical systems it is often the case that differentiable topological information can be used to study qualitative and quantitative properties of the system. This paper deals with the problem of determining the periods (of the periodic points) of a class of C^1 self-maps given the homology class of the map. In order to state our main results we need some preliminary notation and definitions.

Let $f: X \to X$ be a continuous map. A *fixed point* of f is a point x of X such that $f(x) = x$. Denote the totality of fixed points by $\mathrm{Fix}(f)$. The point $x \in X$ is *periodic with period* m if $x \in \mathrm{Fix}(f^m)$ but $x \notin \mathrm{Fix}(f^k)$ for all $k = 1, \dots, m-1$. Let $\mathrm{Per}(f)$ denote the set of all periods of periodic points of f.

Let M be a compact manifold of dimension n. A continuous map $f: M \to M$ induces endomorphisms $f_{*k}: H_k(M; \mathbb{Q}) \to H_k(M; \mathbb{Q})$ (for $k = 0, 1, \dots, n$) on the rational homology groups of M. The *Lefschetz number* of f is defined by

$$L(f) = \sum_{k=0}^{n} (-1)^k \mathrm{trace}(f_{*k}).$$

By the renowned Lefschetz fixed point theorem: if $L(f) \neq 0$ then f has fixed points (see, for instance, [B]). Of course, we can consider the Lefschetz number of f^m but (in general) it is not true that if $L(f^m) \neq 0$ then f has periodic points of period m. As it is well-known a fixed point of f^m need not have period m, so it will be useful to have a method for detecting the difference

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