Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Chaos, Solitons and Fractals 28 (2006) 788–792
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Relations between distributional, Li–Yorke and ω chaos ☆

Juan Luis García Guirao [a,*], Marek Lampart [b]

[a] Departamento de Matemática Aplicada y Estadística, Universidad Politécnica de Cartagena, C/Paseo Alfonso XIII, 30203-Cartagena (Región de Murcia), Spain
[b] Mathematical Institute at Opava, Silesian University at Opava, Na Rybníčku 1, 746 01 Opava, Czech republic

Accepted 5 August 2005

Abstract

The forcing relations between notions of distributional, Li–Yorke and ω chaos were studied by many authors. In this paper we summarize all known connections between these three different types of chaos and fulfill the results for general compact metric spaces by the construction of a selfmap on a compact perfect set which is ω chaotic, not distributionally chaotic and has zero topological entropy.

1. Introduction and terminology

Let (X, d) be a compact metric space and $C(X)$ the set of all continuous selfmaps $f: X \to X$. By $f^n(x)$ we denote the nth iteration of x under f. The sequence $\{f^n(x)\}_{n=0}^{\infty}$, where $f^0(x) = x$ and $f^{n+1}(x) = f^n(f(x))$, is called the trajectory of x under f. The set $\omega_f(x)$ of all accumulation points of the trajectory is the ω-limit set of x under f.

We consider three notions of the concept of chaos: distributional, Li–Yorke and ω chaos introduced by [16], [13] and [12], respectively:

1.1. Distributional chaos

For an f in the class $C(X)$, for $x, y \in X$, $t \in \mathbb{R}$ and a positive integer n, let

$$\xi(x, y, n, t) = \#\{i; 0 \leqslant i < n \text{ and } d(f^i(x), f^i(y)) < t\},$$

☆ This research was supported in part by MCYT (Ministerio de Ciencia y Tecnología, Spain) and FEDER (Fondo Europeo de Desarrollo Regional), grants BEC2001-0535 and BFM2002-03512; Fundación Séneca (Comunidad Autónoma de la Región de Murcia), grant 00684-FI-04, and by Grant Agency of the Czech Republic (grant 201/03/H152) and MSM 4781305904 from the Czech Ministry of Education.
* Corresponding author.
E-mail addresses: juan.garcia@upct.es (J.L.G. Guirao), marek.lampart@math.slu.cz (M. Lampart).

0960-0779/$ - see front matter
doi:10.1016/j.chaos.2005.08.005