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Minimal Lefschetz sets of periods for Morse–Smale diffeomorphisms on the *n*-dimensional torus

Juan Luis García Guirao[a] and Jaume Llibre[b]*

[a]*Departamento de Matemática Aplicada y Estadística, Universidad Politécnica de Cartagena, Antiguo Hospital de Marina, 30203 Cartagena (Región de Murcia), Spain;* [b]*Departament de Matemàtiques, Universitat Autònoma de Barcelona, Bellaterra, 08193 Barcelona (Catalonia), Spain*

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Dedicated to Robert Devaney on the occasion of his 60th birthday

We present a complete description of the minimal Lefschetz set of periods for every homological class of Morse–Smale diffeomorphisms defined on the n-dimensional torus for $n = 1, 2, 3, 4$, by using the Lefschetz zeta function. The techniques applied for obtaining these results also work for $n > 4$.

Keywords: periodic point; minimal Lefschetz set of periods; Morse–Smale diffeomorphism

AMS Subject Classification: 58F20

1. Introduction and statement of the main results

We consider discrete dynamical systems given by a self-diffeomorphism f defined on a given compact manifold $\mathbb{M}$. In this setting, usually the periodic orbits play an important role. In dynamical systems, often the topological information can be used to study qualitative and quantitative properties of the system. Perhaps, the best known example in this direction is the results contained in the paper entitled *Period three implies chaos* for continuous self-maps on the interval [17].

For continuous self-maps on compact manifolds, one of the most useful tools for proving the existence of fixed points, or more generally of periodic points, is the Lefschetz fixed point theorem and its improvements, see for instance [2,3,7–10,12,18,20]. The Lefschetz zeta function $\mathcal{Z}_f(t)$ simplifies the study of the periodic points of f. This is a generating function for the Lefschetz numbers of all iterates of f.

In this work, we put our attention in the class of discrete smooth dynamical systems defined by the *Morse–Smale diffeomorphisms on the tori*.

We denote by Diff($\mathbb{M}$) the space of $\mathcal{C}^1$ diffeomorphisms on a compact manifold $\mathbb{M}$. This space is a topological space endowed with the topology of the supremum with respect to f and its differential Df. In this paper, all the diffeomorphisms will be $\mathcal{C}^1$.

We say that two diffeomorphisms $f, g \in$ Diff($\mathbb{M}$) are *topologically equivalent* if and only if there exists a homeomorphism $h : \mathbb{M} \to \mathbb{M}$ such that $h \circ f = g \circ h$. A diffeomorphism f is *structurally stable* if there exists a neighborhood $\mathcal{U}$ of f in Diff($\mathbb{M}$) such that each $g \in \mathcal{U}$

*Corresponding author. Email: jllibre@mat.uab.cat

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