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On the Lefschetz periodic point free continuous self-maps on connected compact manifolds

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ABSTRACT

We characterize the Lefschetz periodic point free self-continuous maps on the following connected compact manifolds: $\mathbb{C}P^n$ the n-dimensional complex projective space, $\mathbb{H}P^n$ the n-dimensional quaternion projective space, $\mathbb{S}^n$ the n-dimensional sphere and $\mathbb{S}^p \times \mathbb{S}^q$ the product space of the p-dimensional with the q-dimensional spheres.

1. Introduction and statement of the main results

We consider the discrete dynamical system $(\mathbb{M}, f)$ where $\mathbb{M}$ is a topological space and $f : \mathbb{M} \to \mathbb{M}$ be a continuous map. A point x is called *fixed* if $f(x) = x$, and *periodic* of *period* k if $f^k(x) = x$ and $f^i(x) \neq x$ if $0 < i < k$. By $\mathrm{Per}(f)$ we denote the *set of periods* of all the periodic points of f.

If $x \in \mathbb{M}$ the set $\{x, f(x), f^2(x), \dots, f^n(x), \dots\}$ is called the *orbit* of the point x. Here f^n means the composition of n times f with itself. To study the dynamics of the map f is to study all the different kind of orbits of f. Of course if x is a periodic point of f of period k, then its orbit is $\{x, f(x), f^2(x), \dots, f^{k-1}(x)\}$, and it is called a *periodic orbit*.

The periodic orbits play an important role in the general dynamics of the system, for studying them we can use topological information. Perhaps the best known example in this direction are the results contained in the seminal paper entitle *Period three implies chaos* for continuous self-maps on the interval, see [15].

Let $\mathbb{M}$ be a connected compact manifold. Our aim would be characterize classes of continuous self-maps f on $\mathbb{M}$ which are *periodic point free*, i.e. for which $\mathrm{Per}(f) = \emptyset$.

There are only two 1-dimensional connected compact manifolds, the interval and the circle. It is well known that any continuous self-map on the interval has fixed points, so there are no periodic point free maps on the interval. The circle

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