TAIWANESE JOURNAL OF MATHEMATICS
Vol. 16, No. 1, pp. 323-334, February 2012
This paper is available online at http://tjm.math.ntu.edu.tw

$\mathcal{C}^1$ SELF-MAPS ON $\mathbb{S}^n$, $\mathbb{S}^n \times \mathbb{S}^m$, $\mathbb{C}\mathrm{P}^n$ AND $\mathbb{H}\mathrm{P}^n$ WITH ALL THEIR PERIODIC ORBITS HYPERBOLIC

Juan Luis García Guirao and Jaume Llibre

Abstract. We study in its homological class the periodic structure of the $\mathcal{C}^1$ self–maps on the manifolds $\mathbb{S}^n$ (the n–dimensional sphere), $\mathbb{S}^n \times \mathbb{S}^m$ (the product space of the n–dimensional with the m–dimensional spheres), $\mathbb{C}\mathrm{P}^n$ (the n–dimensional complex projective space) and $\mathbb{H}\mathrm{P}^n$ (the n–dimensional quaternion projective space), having all their periodic orbits hyperbolic.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

Let $\mathbb{M}$ be topological space and $f : \mathbb{M} \to \mathbb{M}$ be a continuous map. A point x is called *fixed* if $f(x) = x$, and *periodic* of *period* k if $f^k(x) = x$ and $f^i(x) \neq x$ if $0 \leq i < k$. By Per(f) we denote the *set of periods* of all the periodic points of f.

If $x \in \mathbb{M}$ the set $\{x, f(x), f^2(x), \ldots, f^n(x), \ldots\}$ is called the *orbit* of the periodic point x. Here f^n means the composition of n times f with itself. To study the dynamics of the map f is to study all the different kind of orbits of f. Of course if x is a periodic point of f of period k, then its orbit is $\{x, f(x), f^2(x), \ldots, f^{k-1}(x)\}$, and it is called a *periodic orbit*.

In this paper we study the periodic dynamics of $\mathcal{C}^1$ self–maps f defined on a given compact manifold $\mathbb{M}$ without boundary. Often the periodic orbits play an important role in the general dynamics of a map, for studying them we can use topological information. Perhaps the best known example in this direction are the results contained in the seminal paper entitle *Period three implies chaos* for continuous self–maps on the interval, see [11].

Received October 28, 2010, accepted November 1, 2010.
Communicated by Yingfei Yi.
2010 *Mathematics Subject Classification*: 37C05, 37C25, 37C30.
Key words and phrases: Hyperbolic periodic point, Period, Lefschetz zeta function, Lefschetz number, Sphere, Complex projective space, Quaternion projective space.
The first author was partially supported by MCYT/FEDER grant number MTM2008-03679/MTM, Fundación Séneca de la Región de Murcia, grant number 08667/PI/08 and JCCM (Junta de Comunidades de Castilla-La Mancha), grant number PEII09-0220-0222.
The second author was partially supported by MCYT/FEDER grant number MTM2008-03437, by CICYT grant number 2009SGR 410 and ICREA Academia.