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On the set of periods for the Morse–Smale diffeomorphisms on the disc with N holes

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Let $\mathbb{D}_n$ be the two-dimensional disc with n holes. We assume that $\mathbb{D}_n$ is compact. For every homological class of Morse–Smale diffeomorphisms on $\mathbb{D}_n$ without periodic points in its boundary, we provide an algorithm for characterizing its minimal set of Lefschetz periods. We give the complete classification of these sets of periods for $n = 1, 2, 3, 4, 5$. The main tool used for this characterization is the Lefschetz zeta function.

Keywords: periodic point; minimal set of Lefschetz periods; Morse–Smale diffeomorphism; disc with n holes; Lefschetz number

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1. Introduction

We deal with discrete dynamical systems, defined by a diffeomorphism f on a given compact manifold $\mathbb{M}$ with or without boundary. The periodic orbits play an important role in the dynamics of these systems. In discrete dynamical systems often the topological information can be used for studying qualitative and quantitative properties of the system. The best known example in this direction is the result of the seminal paper entitled *Period three implies chaos* for continuous self-maps on the interval, see [17].

For continuous self-maps on compact manifolds, one of the most useful tools for proving the existence of fixed points, or more generally of periodic points, is the Lefschetz fixed point theorem and its improvements, see for instance [1,2,6–9,11,18,22]. The Lefschetz zeta function $\mathcal{Z}_f(t)$ simplifies the study of the periodic points of f. This function is a generating function for the Lefschetz numbers of all iterates of f.

In this paper we study the class of discrete smooth dynamical systems defined by the *Morse–Smale diffeomorphisms on the closed disc with holes without periodic points in its boundary*. First we recall the definition of a Morse–Smale diffeomorphism.

We denote by Diff($\mathbb{M}$) the space of $\mathcal{C}^1$ diffeomorphisms on a compact Riemannian manifold $\mathbb{M}$. This space is a topological space endowed with the topology of the supremum with respect to f and its differential Df, see for more detail [15]. In this work all the diffeomorphisms will be $\mathcal{C}^1$.

Let f^m be the mth iterate of $f \in$ Diff($\mathbb{M}$). A point $x \in \mathbb{M}$ is a *non-wandering point* of f if for any neighbourhood $\mathcal{U}$ of x there is a positive integer m such that $f^m(\mathcal{U}) \cap \mathcal{U} \neq \emptyset$. We denote by $\Omega(f)$ the set of non-wandering points of f.

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