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Qualitative Theory of Dynamical Systems

Periodic Structure of Transversal Maps on $\mathbb{C}\mathrm{P}^n$, $\mathbb{H}\mathrm{P}^n$ and $\mathbb{S}^p \times \mathbb{S}^q$

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Abstract A $\mathcal{C}^1$ map $f : \mathbb{M} \to \mathbb{M}$ is called transversal if for all $m \in \mathbb{N}$ the graph of f^m intersects transversally the diagonal of $M \times M$ at each point (x, x) being x a fixed point of f^m. Let $\mathbb{C}\mathrm{P}^n$ be the n-dimensional complex projective space, $\mathbb{H}\mathrm{P}^n$ be the n-dimensional quaternion projective space and $\mathbb{S}^p \times \mathbb{S}^q$ be the product space of the p-dimensional with the q-dimensional spheres, $p \neq q$. Then for the cases $\mathbb{M}$ equal to $\mathbb{C}\mathrm{P}^n$, $\mathbb{H}\mathrm{P}^n$ and $\mathbb{S}^p \times \mathbb{S}^q$ we study the set of periods of f by using the Lefschetz numbers for periodic points.

Keywords Periodic point · Period · Transversal map · Lefschetz zeta function · Lefschetz number · Lefschetz number for periodic point · Sphere · Complex projective space · Quaternion projective space

Mathematics Subject Classification (1991) Primary 37C05 · 37C25 · 37C30

1 Introduction and Statement of the Main Results

We consider the discrete dynamical system $(\mathbb{M}, f)$ where $\mathbb{M}$ is a topological space and $f : \mathbb{M} \to \mathbb{M}$ be a continuous map. A point x is called *fixed* if $f(x) = x$, and *periodic* of *period* k if $f^k(x) = x$ and $f^i(x) \neq x$ if $0 \leq i < k$. By $\mathrm{Per}(f)$ we denote the *set of periods* of all the periodic points of f.

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