PERIODS OF CONTINUOUS MAPS ON CLOSED SURFACES

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ABSTRACT. The objective of the present work is to present information on the set of periodic points of a continuous self-map on a closed surface which can be obtained using the action of this map on homological groups of the closed surface.

1. Introduction. Periodic orbits play an important role in the description of the dynamics of a map; through their study, we can use topological information. Perhaps the best known result in this direction is that contained in the seminal paper entitled, "Period three implies chaos," for continuous self-maps on the interval, see [8], and of course, Sharkovskii's famous theorem [10] describing the full set of periods of continuous self-maps on the interval. In addition, the set of periods of continuous self-maps on the circle has been characterized, see for instance, [1].

The interval and the circle are unique compact manifolds of dimension one. After studying the set of periods of self-continuous maps on these manifolds, the next natural step is to begin study of continuous self-maps on compact manifolds of dimension two, i.e., on closed surfaces. This is the main goal of this paper, and we shall use the homological information of these maps to provide information regarding the periods of their periodic orbits.

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