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TOPOLOGICAL ENTROPY AND PERIODS OF SELF–MAPS ON COMPACT MANIFOLDS

JUAN LUIS GARCÍA GUIRAO AND JAUME LLIBRE

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Abstract. Let $(\mathbb{M}, f)$ be a discrete dynamical system induced by a self–map f defined on a smooth compact connected n–dimensional manifold $\mathbb{M}$. We provide sufficient conditions in terms of the Lefschetz zeta function in order that: (1) f has positive topological entropy when f is $\mathcal{C}^\infty$, and (2) f has infinitely many periodic points when f is $\mathcal{C}^1$ and $f(\mathbb{M}) \subseteq \mathrm{Int}(\mathbb{M})$. Moreover, for the particular manifolds $\mathbb{S}^n$, $\mathbb{S}^n \times \mathbb{S}^m$, $\mathbb{C}P^n$ and $\mathbb{H}P^n$ we improve the previous sufficient conditions.

1. Introduction and statement of the main results

Along this work we study discrete dynamical systems $(\mathbb{M}, f)$ induced by a continuous self–map f where $\mathbb{M}$ is a smooth compact connected n–dimensional manifold possibly with boundary. Frequently algebraic information of a given discrete dynamical system provides qualitative or quantitative results on their orbits. Here we use algebraic properties of the Lefschetz zeta function $\mathcal{Z}_f(f)$ associated to f, which is of the form $P(t)/Q(t)$ where $P(t)$ and $Q(t)$ are polynomials, to provide information on the positivity of the topological entropy, and on the infiniteness of the set of periodic points of the system.

Recall that a point $x \in \mathbb{M}$ is *periodic of period* n if $f^n(x) = x$ and $f^k(x) \neq x$ for $k = 1, \ldots, n-1$. On the other hand, roughly speaking, the *topological entropy* of a system $h(f)$ is a non–negative real number (possibly infinite) which measures how much f mixes up the phase space $\mathbb{M}$. When $h(f)$ is positive the dynamics of the system is said to be *complex* and the positivity of $h(f)$ is used as a measure of

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