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Periods of continuous maps on some compact spaces

Juan Luis García Guirao[a] and Jaume Llibre[b]

[a]Departamento de Matemática Aplicada y Estadística, Universidad Politécnica de Cartagena, Cartagena, Spain; [b]Departament de Matemàtiques, Universitat Autònoma de Barcelona, Barcelona, Spain

ABSTRACT

The objective of this paper is to provide information on the set of periodic points of a continuous self-map defined in the following compact spaces: $\mathbb{S}^n$ (the n-dimensional sphere), $\mathbb{S}^n \times \mathbb{S}^m$ (the product space of the n-dimensional with the m-dimensional spheres), $\mathbb{C}P^n$ (the n-dimensional complex projective space) and $\mathbb{H}P^n$ (the n-dimensional quaternion projective space). We use as main tool the action of the map on the homology groups of these compact spaces.

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1. Introduction

Let $f : \mathbb{X} \to \mathbb{X}$ be a continuous map on a compact space $\mathbb{X}$. A point $x \in \mathbb{X}$ is *periodic of period* n if $f^n(x) = x$ and $f^k(x) \neq x$ for $k = 1, \dots, n-1$. We denote by $\mathrm{Per}(f)$ the *set of periods* of all periodic points of f. The aim of the present paper is to provide some information on $\mathrm{Per}(f)$ for some compact spaces. More precisely, we shall present results for the spaces $\mathbb{X} \in \Delta$, where Δ is the set formed by the spaces: $\mathbb{S}^n$ (the n-dimensional sphere), $\mathbb{S}^n \times \mathbb{S}^m$ (the product space of the n-dimensional with the m-dimensional spheres), $\mathbb{C}P^n$ (the n-dimensional complex projective space) and $\mathbb{H}P^n$ (the n-dimensional quaternion projective space).

The statements of our main results are the following ones.

Theorem 1: *Let f be a continuous self-map on $\mathbb{S}^n$ of degree D. Then the following statements hold.*

(a) *If n is even and $D = -1$, then $\mathrm{Per}(f) \cap \{1, 2\} \neq \emptyset$.*
(b) *If n is odd and $D \neq 1$, then $\mathrm{Per}(f) \cap \{1\} \neq \emptyset$.*

Theorem 2: *Let f be a continuous self-map on $\mathbb{S}^n \times \mathbb{S}^n$ of degree D, and let $f_{*n} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $a, b, c, d \in \mathbb{Z}$, be the action of f on the nth homology group $H_n(\mathbb{S}^n \times \mathbb{S}^n, \mathbb{Q}) \approx \mathbb{Q} \oplus \mathbb{Q}$. Then the following statements hold.*

(a) *Assume n is even.*
  (a.1) *If $1 + a - d + D \neq 0$, then $\mathrm{Per}(f) \cap \{1\} \neq \emptyset$.*

CONTACT Juan Luis García Guirao juan.garcia@upct.es