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On the periods of a continuous self-map on a graph

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Abstract
Let G be a graph and f be a continuous self-map on G. We present new and known results (from another point of view) on the periods of the periodic orbits of f using mainly the action of f on its homology, or the shape of the graph G.

Keywords Topological graph · Discrete dynamical systems · Lefschetz numbers · Lefschetz zeta function · Periodic point · Period

Mathematics Subject Classification 37E25 · 37C25 · 37C30

1 Introduction and statement of the main results

A *discrete dynamical system* (G, f) is formed by a continuous map $f : G \to G$, where G is a topological space.

A point $x \in G$ is *periodic* of *period* k if $f^k(x) = x$ and $f^i(x) \neq x$ if $0 < i < k$. If $k = 1$, then x is called a *fixed point*. Per(f) denotes the *set of periods* of all the periodic points of f.

The *orbit* of the point $x \in G$ is the set $\{x, f(x), f^2(x), \ldots, f^n(x), \ldots\}$, whereby f^n we denote the composition of f with itself n times. To knowledge the behavior of all different kinds of orbits of f is to study *the dynamics of the map* f.

Many times the periodic points play an important role for understanding the dynamics of a discrete dynamical system. One of the best known results in this direction is the paper *Period three implies chaos* for continuous interval maps, see Li and Yorke (1975).

Here, a *graph* G is a compact connected space containing a finite set V, such that $G\backslash V$ has finitely many open connected components, each one homeomorphic to the interval $(0, 1)$,

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