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ORIGINAL PAPER

On the dynamics of the rigid body with a fixed point: periodic orbits and integrability

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Abstract The aim of the present paper is to study the periodic orbits of a rigid body with a fixed point and quasi-spherical shape under the effect of a Newtonian force field given by different small potentials. For studying these periodic orbits, we shall use averaging theory. Moreover, we provide information on the $\mathcal{C}^1$-integrability of these motions.

Keywords Rigid body with a fixed point · Periodic orbits · Integrability · Averaging theory

1 Introduction and statement of the main results

The dynamics of rigid bodies, in its different formulations Eulerian, Lagrangian, and Hamiltonian, has been extensively studied in the classic literature; see, for instance [8], for a classic treatment of these topics or [1, 2], and [10] for a more modern approach.

The main objectives in the study of the motion of a rigid body with a fixed point are:

1. To state the equilibria and their stabilities in rigid bodies with a fixed point.
2. To state the periodic solutions, bifurcations, and chaos of its motion.
3. To analyze the integrability and to state the first integrals for the problem.

Our main aim in this work is to study the periodic orbits of a rigid body with a fixed point under a Newtonian gravitational field using averaging theory. As a corollary, we also obtain information about $\mathcal{C}^1$ nonintegrability.

It is well known, see for instance [3, p. 57], that the motion of a rigid body with a fixed point is described by the Hamiltonian equations associated to the Hamiltonian

$$\mathcal{H} = \frac{(G^2 - L^2)}{2}\left(\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B}\right) + \frac{L^2}{2C} + U(k_1, k_2, k_3) \tag{1}$$

with

$$k_1 = \left(\frac{H}{G}\sqrt{1-\left(\frac{L}{G}\right)^2} + \frac{L}{G}\sqrt{1-\left(\frac{H}{G}\right)^2}\cos g\right)\sin l$$

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