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ORIGINAL PAPER

Periodic solutions induced by an upright position of small oscillations of a sleeping symmetrical gyrostat

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Abstract The aim of this paper is to provide sufficient conditions for the existence of periodic solutions emerging from an upright position of small oscillations of a sleeping symmetrical gyrostat with equations of motion

$$\ddot{x} + \alpha \dot{y} - \beta x = \varepsilon F_1(t, x, \dot{x}, y, \dot{y}),$$
$$\ddot{y} - \alpha \dot{x} - \beta y = \varepsilon F_2(t, x, \dot{x}, y, \dot{y}).$$

being α and β parameters satisfying $\Delta = \alpha^2 - 4\beta > 0$ and $\beta - \frac{\alpha^2}{2} \pm \frac{\alpha\sqrt{\Delta}}{2} < 0$, ε a small parameter and, F_1 and F_2 smooth periodic maps in the variable t in resonance p:q with some of the periodic solutions of the system for $\varepsilon = 0$, where p and q are positive integers relatively prime. The main tool used is the averaging theory.

Keywords Periodic solution · Symmetrical gyrostat · Averaging theory

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1 Introduction and statement of the main results

The equations of motion of a sleeping gyrostat on the upright position under the influence of small periodic momenta are

$$\begin{aligned} \ddot{x} + \alpha \dot{y} - \beta x &= \varepsilon F_1(t, x, \dot{x}, y, \dot{y}), \\ \ddot{y} - \alpha \dot{x} - \beta y &= \varepsilon F_2(t, x, \dot{x}, y, \dot{y}), \end{aligned} \tag{1}$$

for more details on them see Appendix 2 or papers [6] and [9]. Here the dot denotes derivative with respect to the time t. The parameter ε is small and the smooth functions F_1 and F_2 define the perturbed torques which, in general, are periodic functions in the variable t and in resonance p:q with some of the periodic solutions of the sleeping symmetrical gyrostat for $\varepsilon = 0$, being p and q positive integers relatively prime.

Recall that a gyrostat is a mechanical system S composed by a rigid solid S_1 to which other bodies S_2 are connected; these other bodies may be variable or rigid, but the key property is that they must not be rigidly connected to S_1, so that the movements of S_2 with respect to S_1 do not modify the distribution of mass within the compound system S.