PERIODIC ORBITS OF HAMILTONIAN SYSTEMS: APPLICATIONS TO PERTURBED KEPLER PROBLEMS

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Abstract. We provide for a class of Hamiltonian systems in the action–angle variables sufficient conditions for showing the existence of periodic orbits. We expand this result to the study of the existence of periodic orbits of perturbed spatial Keplerian Hamiltonians with axial symmetry. Finally, we apply these general results for finding periodic orbits of the Matese–Whitman Hamiltonian, of the spatial anisotropic Hamiltonian and of the spatial generalized van der Waals Hamiltonian.

1. Introduction and statement of the main results

We consider the following class of Hamiltonians in the action–angle variables

$$\mathcal{H}(I_1,...,I_n,\theta_1,...,\theta_n) = \mathcal{H}_0(I_1) + \varepsilon\mathcal{H}_1(I_1,...,I_n,\theta_1,...,\theta_n), \tag{1}$$

where ε is a small parameter.

As usual the *Poisson bracket* of the functions $f(I_1,...,I_n,\theta_1,...,\theta_n)$ and $g(I_1,...,I_n,\theta_1,...,\theta_n)$ is

$$\{f,g\} = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial\theta_i}\frac{\partial g}{\partial I_i} - \frac{\partial f}{\partial I_i}\frac{\partial g}{\partial\theta_i}\right).$$

The next result provides sufficient conditions for computing periodic orbits of the Hamiltonian system associated to the Hamiltonian (1).

Theorem 1. *We define*

$$\langle\mathcal{H}_1\rangle = \frac{1}{2\pi}\int_0^{2\pi}\mathcal{H}_1(I_1,...,I_n,\theta_1,...,\theta_n)d\theta_1,$$

and we consider the differential system

$$\begin{aligned}\frac{dI_i}{d\theta_1} &= \varepsilon\frac{\{I_i,\langle\mathcal{H}_1\rangle\}}{\mathcal{H}_0'(\mathcal{H}_0^{-1}(h^*))} = \varepsilon f_{i-1}(I_2,...,I_n,\theta_2,...,\theta_n) \qquad i=2,...,n,\\ \frac{d\theta_i}{d\theta_1} &= \varepsilon\frac{\{\theta_i,\langle\mathcal{H}_1\rangle\}}{\mathcal{H}_0'(\mathcal{H}_0^{-1}(h^*))} = \varepsilon f_{i+n-2}(I_2,...,I_n,\theta_2,...,\theta_n) \qquad i=2,...,n,\end{aligned} \tag{2}$$

Key words and phrases. Periodic orbits, perturbed Kepler problem, the Matese–Whitman Hamiltonian, spatial anisotropic Kepler problem, spatial generalized van del Waals Hamiltonian.

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