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ORIGINAL PAPER

Periodic orbits of a perturbed 3-dimensional isotropic oscillator with axial symmetry

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Abstract We study the periodic orbits of a generalized Yang–Mills Hamiltonian $\mathcal{H}$ depending on a parameter β. Playing with the parameter β we are considering extensions of the Contopoulos and of the Yang–Mills Hamiltonians in a 3-dimensional space. This Hamiltonian consists of a 3-dimensional isotropic harmonic oscillator plus a homogeneous potential of fourth degree having an axial symmetry, which implies that the third component N of the angular momentum is constant. We prove that in each invariant space $\mathcal{H} = h > 0$ the Hamiltonian system has at least four periodic solutions if either $\beta < 0$, or $\beta = 5+\sqrt{13}$; and at least 12 periodic solutions if $\beta > 6$ and $\beta \neq 5+\sqrt{13}$. We also study the linear stability or instability of these periodic solutions.

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1 Introduction and statement of the main results

In this paper we study the periodic solutions of a generalized Yang–Mills Hamiltonian (see [16,19]), which consists of a 3-dimensional (or simply 3D) isotropic harmonic oscillator perturbed by a homogeneous potential of fourth degree

$$\mathcal{H} = \frac{1}{2}\left(p_1^2 + p_2^2 + p_3^2\right) + \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right) + \varepsilon\left((x_1^2 + x_2^2)^2 + \beta(x_1^2 + x_2^2)x_3^2\right). \tag{1}$$

Of course ε is a small parameter. This perturbation exhibits an axial symmetry with respect to the x_3-axis which depends on one real parameters β, and thus its study can be reduced to a family of Hamiltonian systems with 2 degrees of freedom fixing the third component of the angular momentum. In [19] two Hamiltonians are studied whose motions take place in the plane (x_1, x_2), one with a cubic potential and another with a quartic one, while we study a Hamiltonian in the space (x_1, x_2, x_3) whose motion takes place outside the plane (x_1, x_2) when the third component of the angular