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A note on the equilibria of an economic model with local competition "à la Cournot"

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ABSTRACT

Guirao and Rubio (2010) [12] introduce an economic model, which generalizes the classical duopoly of Cournot type, where the competitors are located around a circle or a line and each firm competes "à la Cournot" with its right and left neighboring. For the case of having three and four players, we describe completely the bifurcations of equilibria in terms of the production costs of each firm and we study the stability of them. Moreover, for the case of four players we provide some information on the two-periodic orbits of the system.

1. Introduction and statement of the main result

Cournot duopoly was introduced by Cournot [1] who is considered one of the forerunners of the modern microeconomics. The process consists of two firms which produce an identical good and which compete for the market. In each step of the process the firms decide the amount of product to be introduced in the market, and for making this decision both firms should know the amount of product introduced in the market in the previous step by the rival firm. This economic process is mathematically modeled by the following two-dimensional discrete dynamical system

$$F(x, y) = (g(y), f(x)) \tag{1}$$

where f, g are continuous self-maps defined on a compact interval which can be considered, without loss of generality, by normalization $[0, 1]$. The maps f and g are called the *reaction functions* and determine the decisions made by the firms.

Note that if firm A put on the market at the beginning of the game α_0 product and firm B put β_0, in the next step of the game firm A will produce $g(\beta_0)$, i.e. an amount of product which directly depends on the production level of the firm B in the previous step, on the other hand, firm B will produce $f(\alpha_0)$ and so on. Therefore, whole of the process is governed by the dynamics of the discrete system (1) which strongly depends on the dynamics of the one-dimensional interval maps f and g.

Duopoly is an intermediate situation between monopoly and perfect competition, and analytically is a more complicated case. The reason for this is that oligopolist must consider not only the behaviors of the customers, but also those of the competitors and their reactions. Thus this model has been studied in the literature from different points of view; see for instance [2–9] or [10].

While dynamic properties of duopolies have been extensively studied, adjustment dynamics in Cournot processes with more than two players have received much less attention as a consequence of the difficulties which appear for studying discrete dynamical systems with a dimension higher than two. The direct generalization of the Cournot duopoly situation is

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