



# Darboux integrability and dynamics of the Basener–Ross population model

Faruk Güngör<sup>1</sup> · Jaume Llibre<sup>2</sup> · Chara Pantazi<sup>3</sup>

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## Abstract

We deal with the Basener and Ross model for the evolution of human population in Easter island. We study the Darboux integrability of this model and characterize all its global dynamics in the Poincaré disc, obtaining 15 different topological phase portraits.

**Keywords** Basener–Ross population model · Quadratic system · Poincaré disc · Darboux integrability · Darboux invariant

**Mathematics Subject Classification** Primary 34C05 · 34C23

## 1 Introduction and statement of the main results

In order to explore the evolution of ecosystems an isolated island is a good laboratory due to the total absence of external distortion factors like migration. Basener and Ross, see equation (5) in [3], proposed the following model for the evolution of human population in Easter island

$$\dot{x} = x(1 - y), \quad \dot{y} = (h - 1)y^2 + (1 - c)y + \frac{c}{k}x, \quad (1)$$

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✉ Faruk Güngör  
gungorf@itu.edu.tr

Jaume Llibre  
jllibre@mat.uab.cat

Chara Pantazi  
chara.pantazi@upc.edu

<sup>1</sup> Department of Mathematics, Faculty of Science and Letters, Istanbul Technical University, 34469 Istanbul, Turkey

<sup>2</sup> Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

<sup>3</sup> Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya, (EPSEB), Av. Doctor Marañón, 44-50, 08028 Barcelona, Spain