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Non–Equivalence Between the Melnikov and the Averaging Methods for Nonsmooth Differential Systems

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Abstract
It is known that for smooth differential systems in the plane $\mathbb{R}^2$ the Melnikov and the averaging methods for studying the limit cycles produce the same results. Here we prove that this is not the case for nonsmooth differential systems in the plane. More precisely, we prove that the linear system $\dot{x} = y$, $\dot{y} = -x$, can produce at most 5 crossing limit cycles using the averaging theory of first order and also produce at most 5 crossing limit cycles using the averaging theory of second order, when it is perturbed by discontinuous piecewise polynomials of two pieces separated by the cubic curve $y = x^3$, and having in each piece a quadratic polynomial differential system. While using the Melnikov theory up to second order these discontinuous piecewise differential systems already produce 7 crossing limit cycles having in each piece a linear polynomial differential system. Note that the class of the linear polynomial differential systems is contained into the class of quadratic polynomial differential systems.

Keywords Limit cycles · The averaging method · Discontinuous piecewise differential systems

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