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On Uniqueness of Limit Cycles in General Bogdanov–Takens Bifurcation

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In this paper, we present a complete study of the well-known Bogdanov–Takens bifurcation and give a rigorous proof for the uniqueness of limit cycles.

Keywords: Bogdanov–Takens bifurcation; limit cycle.

1. Introduction

General theory of Bogdanov–Takens bifurcation is well known. It was established by Bogdanov [1981] and then introduced in many books, see [Carr, 1982; Chow & Hale, 1996; Chow *et al.*, 1994; Guckenheimer & Holme, 1983; Kuznetsov, 1995, 1998, 2004] and [Han & Gu, 2001; Han & Zhu, 1994; Zhang *et al.*, 1997; Zhao *et al.*, 2011]. However, to our knowledge, we have not found a really satisfactory and complete study in any of the current works in English. For example, the uniqueness of limit cycles in homoclinic bifurcation was not considered in all of these works in English. The problem

We organize the paper as follows. In Sec. 2, we state some preliminary theorems which are useful in studying Bogdanov–Takens bifurcation. The preliminary theorems on Hopf and homoclinic bifurcations were obtained after the paper of Bogdanov [1981].

was first found and solved in the paper [Luo *et al.*, 1992]. One can find a relatively complete study on it in books [Han & Gu, 2001; Han & Zhu, 1994; Zhang *et al.*, 1997; Zhao *et al.*, 2011] with different details. These books are all in Chinese. The aim of this paper is to provide a complete study with all details on the well-known Bogdanov–Takens bifurcation.

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