




Abelian Integrals and Non-generic Turning Points

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Abstract

In this paper we initiate the study of the Chebyshev property of Abelian integrals generated by a non-generic turning point in planar slow-fast systems. Such Abelian integrals generalize the Abelian integrals produced by a slow-fast Hopf point (or generic turning point), introduced in Dumortier et al. (Discrete Contin Dyn Syst Ser S 2(4):723–781, 2009), and play an important role in studying the number of limit cycles born from the non-generic turning point.

Keywords Abelian integrals · Chebyshev systems · Planar turning points

Mathematics Subject Classification 34E15 · 34E17

1 Introduction

In this work we are concerned with the study of the Abelian integrals

$$I_j(h) = \int_{\gamma_h} e^{-2n\bar{y}} \bar{x}^{2n+2j+1} d\bar{y}, \quad n \in \mathbb{N}, \quad j = -n, 0, 1, 2, \dots$$

and with their derivatives

$$\frac{d}{dh} I_j(h) = -\frac{2n+2j+1}{4n^2} \bar{I}_j(h) \quad \text{where} \quad \bar{I}_j(h) = \int_{\gamma_h} \bar{x}^{2j+1} d\bar{y}. \quad (1)$$

A proof of (1) is given in Sect. 2. Here γ_h , with $h \in]0, 1[$, denotes the oval surrounding the origin described by the level of energy $\{H(\bar{x}, \bar{y}) = h\}$ where

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