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Bounded and dissipative solutions of the Bouc-Wen model for hysteretic structural systems

Fayçal Ikhouane, Víctor Mañosa, José Rodellar

***Abstract*— The aim of this paper is to give the conditions on the hysteretic Bouc-Wen model so that (i) it has the property of being bounded input bounded output (BIBO), and (ii) it dissipates energy (or is passive). This study leads to a classification of the possible BIBO and dissipative (or passive) Bouc-Wen models.**

I. Introduction

To describe the behavior of hysteretic processes several mathematical models have been proposed [11]: the Duhem model uses the property that a hysteretic system's output changes its character when the input changes direction; the Ishlinskii hysteresis operator has been proposed as a model for plasticity–elasticity and the Preisach model has been used for the modelling of electromagnetic hysteresis . A survey of the mathematical models for hysteresis may be found in [6]. Most works devoted to controlling systems with a continuous hysteresis have used the backlash model (see for example [9], [10]). As noted as early as in [5]: ”Use of backlash to model [a magnetic] hysteresis element misrepresent the behavior about the origin, does not properly account for saturation, and introduces a dead zone that does not exist”.

The objective of the present paper is to introduce an alternative simple model of a smooth hysteresis known as the Bouc-Wen model. Proposed in 1976, this model (a first-order nonlinear differential equation) has been used experimentally mainly in wood joints and structural systems (see [1] for example) and has remained largely unknown for the wide control community due mainly to the absence of an analytical study of this model. The aim of this paper is to fill this gap by giving the conditions on the Bouc-Wen model so that it holds the property of being bounded input bounded output (BIBO) and, moreover, it dissipates energy.

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F. Ikhouane and J. Rodellar are with the Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, C/ Jordi Girona 1-3, 08034 - Barcelona, Spain faycal.ikhouane@upc.es, jose.rodellar@upc.es

V. Mañosa is with the Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, C/ Colom 1, 08222 - Terrassa, Spain victor.manosa@upc.es

II. BIBO Bouc-Wen model properties

A. The model

Consider a physical system with a hysteretic component that can be represented by a map $x(t) \mapsto \Phi_s(x(t),t)$, which is referred to as the “true” hysteresis. The so-called Bouc-Wen model [12] represents the true hysteresis in the following form:

$$\Phi_{BW}(x,t) = \alpha k x(t) + (1-\alpha) D k z(t), \tag{1}$$

$$\dot{z} = D^{-1}\left(A\dot{x} - \beta|\dot{x}|\,|z|^{n-1}z - \gamma\dot{x}|z|^{n}\right), \tag{2}$$

where $\dot{z}$ denotes the time derivative, and $n>1$, $D>0$, $k>0$ and $0<\alpha<1$ are parameters. The limit cases $n=1$, $\alpha=0$, $\alpha=1$ are treated in Appendix C.

This model was originally developed in the context of mechanical systems in which x is a displacement and Φ is a restoring force. It represents the hysteretic force $\Phi_s(x,t)$ as the superposition of an elastic component αkx and a purely hysteretic component $(1-\alpha)kDz$, in which $D>0$ is the yield constant displacement and $\alpha \in (0,1)$ is the post to pre-yielding stiffness ratio. The hysteretic part involves a nondimensional auxiliary variable z which is the solution of the nonlinear first order differential equation (2). In this equation, A, β and γ are nondimensional parameters which control the shape and the size of the hysteresis loop, while n is a scalar that governs the smoothness of the transition from elastic to plastic response.

B. Problem statement

This study lies in the experimentally based premise that a true physical hysteretic element is BIBO, which means that, for any bounded input signal $x(t)$, the hysteretic response is also bounded. Thus the Bouc–Wen model Φ_{BW} should keep the BIBO property in order to be considered an adequate candidate to model real physical systems. Appendix A gives an example of a set of parameters A, β, γ, n such that, for a particular bounded input $x(t)$, the corresponding output $\Phi_{BW}(x(t),t)$ given by the Bouc-Wen model (1)–(2) is unbounded. This means that, with this set of parameters, the Bouc-Wen model cannot describe the true hysteresis. This motivates the following problem:

Given the parameters $0<\alpha<1$, $k>0$, $D>0$, A, β, γ and $n>1$, find the set of initial conditions $z(0)$ for which the Bouc-Wen model (1)-(2) is BIBO. Note that when this set is empty, this means that the Bouc-Wen model is not BIBO. The solution of this problem will lead to classify different