## A RESTRICTED VERSION OF HILBERT'S 16TH PROBLEM FOR QUADRATIC VECTOR FIELDS

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ABSTRACT. The restricted version of Hilbert's 16th problem for quadratic vector fields requires an upper estimate of the number of limit cycles through a vector parameter that characterizes the vector fields considered and the limit cycles to be counted. In this paper we give an upper estimate of the number of limit cycles of quadratic vector fields " $\sigma$ -distant from centers and  $\kappa$ -distant from singular quadratic vector fields" provided that the limit cycles are " $\delta$ -distant from singular points and infinity".

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## 1. INTRODUCTION AND STATEMENT OF THE MAIN RESULT

Hilbert's 16th problem asks (see [6]): what may be said about the number and location of limit cycles of a polynomial vector field of degree n in the real plane? The main contributions in this direction were the works of Écalle [5] and Ilyashenko [8] who proved that any polynomial vector field has finitely many limit cycles, and also the work of Llibre and Rodríguez [13] who showed that any finite location of limit cycles is realized by a polynomial vector field of a convenient degree. But the complete answer to Hilbert's 16th problem is unknown even for n = 2. Even the existence of an uniform upper bound of the number of limit cycles for quadratic vector fields (polynomial vector fields of degree 2) is not yet proved. Limit cycles of a quadratic vector field may surround only one singular point, and it is of type focus (for more details see [14]). Moreover, quadratic vector fields have at most two foci (see again [14]). Limit cycles surrounding the same singular point form a nest. Recently Zhang Pingguang [15], [16], [17] proved that only one nest of quadratic vector field may have more than one limit cycle.

The restricted version of Hilbert's 16th problem for quadratic vector fields allows us to introduce a vector parameter that characterizes the vector field and the limit

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