
O problema do centro-foco para singularidades
nilpotentes no plano

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Abstract

The study of singular points in planar analytic vector fields is a problem almost completely solved. The only case that remains open is the monodromic one, in which the orbits turn around the singularity. In analytic differential systems, if p is a monodromic singular point, then p is either a center or a focus. The center-focus problem consists in determining conditions for distinguishing between a center and a focus.

The main purpose of this work is the investigation of the center-focus problem in analytic differential systems with nilpotent singular points. This problem is still widely studied, since there is no algorithm for such case, comparable to the Lyapunov method for the case of non-degenerate singularities.

We studied two different methods. The first makes use of the normal form theory and deals with the problem in the classic way, splitting it up in two parts: the investigation of the monodromy and the study of the stability. The latter investigates the differential analytic systems with nilpotent singular points as limit of differential systems with non-degenerate singularities.

In order to evaluate the efficiency and understand possible obstructions, we applied the two techniques to concrete families of differential systems.

Key words: The Center-Focus Problem; Lyapunov Constants; Nilpotent Singularities; Monodromy.

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