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LIMIT CYCLES BIFURCATING FROM THE PERIOD ANNULUS OF A UNIFORM ISOCHRONOUS CENTER IN A QUARTIC POLYNOMIAL DIFFERENTIAL SYSTEM

JACKSON ITIKAWA, JAUME LLIBRE

Abstract. We study the number of limit cycles that bifurcate from the periodic solutions surrounding a uniform isochronous center located at the origin of the quartic polynomial differential system

$$\dot{x} = -y + xy(x^2 + y^2), \quad \dot{y} = x + y^2(x^2 + y^2),$$

when perturbed in the class of all quartic polynomial differential systems. Using the averaging theory of first order we show that at least 8 limit cycles bifurcate from the period annulus of the center. Recently this problem was studied by Peng and Feng [9], where the authors found 3 limit cycles.

1. Introduction and statement of main results

One of the main open problems in the qualitative theory of polynomial differential systems in $\mathbb{R}^2$ is the determination of their limit cycles, see for instance [5]. A classical method to produce limit cycles is by perturbing a system which has a center. In this case the perturbed system displays limit cycles that bifurcate either from the center (having the so-called Hopf bifurcation); or from some of the periodic orbits around the center, (see [2, 10] and references therein); or from the graph in the boundary of the period annulus of the center.

Isochronous differential systems constitute a large class of polynomial systems with interesting properties. They arise in many applications. For recent studies on the bifurcation of limit cycles of planar polynomial differential systems having a uniform isochronous center see for instance [4, 6, 7]. In this paper we shall perturb the uniform isochronous center of the quartic polynomial differential system

$$\dot{x} = -y + xy(x^2 + y^2), \quad \dot{y} = x + y^2(x^2 + y^2), \tag{1.1}$$

inside the class of all quartic polynomial differential systems.

Peng and Feng [9] studied the differential system (1.1), showing that under any quartic homogeneous polynomial perturbations, at most 2 limit cycles bifurcate from the period annulus of such system using averaging theory of first order, and

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