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New classes of polynomial maps satisfying the real Jacobian conjecture in $\mathbb{R}^2$

JACKSON ITIKAWA[1] and JAUME LLIBRE[2]

[1]Departamento de Matemática, Universidade Federal de Rondônia, Campus – BR 364, Km 9.5, 76801-059 Porto Velho, RO, Brazil
[2]Department de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

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Abstract: We present two new classes of polynomial maps satisfying the real Jacobian conjecture in $\mathbb{R}^2$. The first class is formed by the polynomials maps of the form $(q(x)-p(y), q(y)+p(x)) : \mathbb{R}^2 \to \mathbb{R}^2$ such that p and q are real polynomials satisfying $p'(x)q'(x) \neq 0$. The second class is formed by polynomials maps $(f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ where f and g are real homogeneous polynomials of the same arbitrary degree satisfying some conditions.

Key words: injective polynomial maps, global center, real Jacobian conjecture, planar Hamiltonian systems

INTRODUCTION

Let $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that its Jacobian never vanishes. The celebrated *real Jacobian conjecture* states that under these conditions F is injective. This conjecture goes back to 1939, see Keller (1939).

In 1994 Pinchuk (1994) found a map $F = (f, g)$ with f and g polynomials of degree 10 and 25 respectively, and with Jacobian strictly positive, such that F is not injective.

Although the real Jacobian conjecture has been proved false by Pinchuk, a considerable number of papers has been devoted to this subject, mainly searching for additional conditions such that the conjecture might hold. The problem of determining if F is injective in the case of its Jacobian to be a non–zero constant, known as the *Jacobian conjecture*, is still open, see Essen (2000) and the references therein for more information.

Correspondence to: Jackson Itikawa
E-mail: itikawa@unir.br
ORCid: https://orcid.org/0000-0002-8268-0016
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